MATHEMATICS: Plane Geometry



Plane Geometry teaches students how to use known facts to verify the truth of additional facts, to solve geometric problems, and to use deductive reasoning for drawing correct conclusions. Students learn to think *naturally, logically,* and *systematically* whenever they encounter a proof to write, a construction to make, or a problem to solve. They are then equipped throughout life to perform such tasks as determining which car is the better buy or identifying truth and flaws in politics.

For this grade level, see also *Precalculus* on p. 191.

Also available: Consumer Mathematics and Business Mathematics on Electives pp. 208-212.

Topical Interest Essays

- Geometry Past and Present; Geometry and the Pyramids
- Euclid, Master of Logic; Geometry and Solomon's Temple
- Archimedes, Greatest Mathematician of Antiquity
- Geometry and the Parthenon; Blaise Pascal, Inventor, Mathematician, Writer
- The Golden Ratio; Leibniz, Universal Genius
- Notre Dame de Paris; Isaac Barrow, Teacher, Friend of Isaac Newton
- Kaleidoscope; Sir Christopher Wren, Mathematician and Architect
- The value of Π; Symmetry in Nature

5 Final Review Exercises Quick Reference & Summaries

- Conversion tables, plane geometry notation and formulas
- Basic mathematics axioms, geometry principles, postulates, and informal statements
- Theorems and corollaries, properties of proportions, transformations
- Constructions, proof methods, factual summaries

Evaluation

- Quizzes (50)
- Tests (8)
- 9-weeks exam (2)
- Semester exam
- Final exam

RED indicates first introduction of content.

Introduction to Geometry

- Geometry defined
- > Principles, informal statements, axioms, postulates
- Fundamental definitions: equal segments, midpoint, trisection
- Angles:
- Equal, bisector
- ➤ Perigon
- Straight angle, adjacent, right, perpendicular lines, acute, obtuse, reflex
- ➤ Oblique
- Complementary, supplementary, vertical
- Measuring angles; degrees, minutes, seconds; protractor; compass
 Triangle:
- Defined, vertices, base, exterior angle
- > Opposite interior angle, median
- Altitude, scalene, isosceles, equilateral, acute, obtuse, right, equiangular, sum of angles
- Polygon:
- Defined
- Base, adjacent sides, diagonal, convex, concave, sum of angles, regular, center
- Circle:
- Defined, center, circumference, diameter, radius
- Chord, arc, semicircle, quadrant
- Congruence:
- Defined
- Corresponding parts
- Constructions:
- Perpendicular bisector, angle bisector, angle
- Perpendicular at a point
- > Perpendicular from a point
- Triangle given three sides, one side and adjacent angles, two sides and included angle

- > Optical illusions, deduction
- Euclid's Elements
- > Numeric applications

Rectilinear Plane Figures

- Demonstrative proof: defined, demonstrated, given, prove, analysis, proof, theorem, corollary
- Triangles congruent by:
- SAS
- → LL
- ASA
- ≻LA
- SSS
- ► HA, SAA, HL
- Triangle sides-angles relationships:
- Isosceles triangle, equilateral, equiangular
- > Exterior-exterior angle, opposite sides-angles
- Auxiliary lines, direct and indirect method of proof
- Parallel lines
- Parallel postulate and corollary
- Transversal formed
- Angles formed, angle relationships
- > Proving lines parallel
- Proving angles equal, supplementary, complementary
- Triangle relationships:
- Sum of angles
- Exterior-opposite interior angles
- Acute angles of right triangle
- > 30°-60°-90°
- > Unequal lines and angles, perpendicular lines, triangles with unequal lines and angles
- Distance defined between two points, two lines, a line and a point
- Parallelograms and quadrilaterals:
- Defined, base, altitude, rhombus, rectangle, square

MATHEMATICS: Plane Geometry cont.

Rectilinear Plane Figures cont.

- > Relationships of sides, angles, diagonals, shapes formed
- Proving a quadrilateral is a parallelogram
- ▶ Segments intersected by parallel lines
- Trapezoid:
- Defined, legs
- ➤ Median
- Altitude
- Isosceles
- Polygons:
- Defined
- > Sum of exterior angles, sum of interior angles
- > Each angle measure, formulas
- Concurrent lines of a triangle:
- > Defined, altitudes
- > Angle bisectors, perpendicular bisectors of sides, medians
- > Proof reasoning methods (critical thinking skills): analytic, synthetic, general method
- Inequality axioms
- Numeric applications

The Circle

- Relationships of equal arcs, central angles, and chords; unequal arcs, central angles, and chords; chord distances from center
- Diameter-chord relationships, perpendicular relationships
- Inscribed and circumscribed polygons
- Tangent lines and relationships, common tangents
- Tangent and intersecting circles, common chord, concentric circles
- Measuring angles and arcs, inscribed angles, semicircles
- Angles formed by combinations of chords, tangents, secants
- Constructions and proofs:
- Reviewed
- ➤ Bisect arc
- Parallel lines
- > Divide a line into *n* equal parts
- > Circumscribe a circle, inscribe a circle, tangent to a circle
- > Circle from various givens, triangle from various givens
- > Locus:
- > Definitions and drawings
- > Fundamental locus theorems:
- > Equidistant and given distances from various points
- > Intersecting and parallel lines
- > Centers of circles tangent to a line, etc.
- Intersecting loci
- General directions for constructions

Proportions & Similar Polygons

- Definitions: ratio, antecedent, consequent, proportion, extremes, means
- > Fourth proportional
- Mean proportional
- > Third proportional, continued proportion
- Fundamental properties: product of means and extremes, writing proportions
- > Like-powers axioms
- Finding a mean proportional

- Transformations: alternation, inversion, addition, subtraction, like powers
- > Proportional segments: by parallel lines, by angle bisectors
- Similar polygons, corresponding sides
- Corresponding angles, ratio of similitude
- > Proving triangles similar aaa, aa, sas, II, sss
- > Proportional line proofs: in triangles, with parallel lines, in right triangles, in circles
- > Pythagorean theorem proved
- Similar polygons:
- Proportional sides, perimeters, diagonals, corresponding lengths
 Similarity of corresponding triangles
- > Construction of proportional segments and polygons: fourth proportional, a given proportional, mean proportional
- Projection
- Numeric applications

Surface Measurement

Defined, equal figures, constant, variable

→ Limit

- Area mensuration formulas for rectangle, square, parallelogram, triangle, trapezoid
- > Area proportions for rectangle, square, parallelogram, triangle, trapezoid
- > Areas of similar triangles and similar polygons
- Pythagorean theorem:
- > By areas of squares, by area of similar polygons
- > Construction of equal non-similar shapes
- > Transforming plane figures
- Numeric applications

Regular Polygons & Circles

- Defined
- > Inscribed and circumscribed, chords, tangents, midpoints
- ► Inscribed and circumscribed circles
- ► Angle at center of n-gon
- > Ratios regarding similar polygons: perimeters, corresponding sides, areas, radii, apothems
- Area formula
 - Circle proportions
 - ▶ Formulas to measure:
 - Circumferences, radii, diameters
 - Pi
 - > Arc length using angles in degrees
 - > Sectors, segments, similar sectors and segments
 - > Constructions: inscribing a square, regular polygons
 - > Numeric applications including 30°-60°-90°, 45°-45°-90°

Trigonometry

- Defined
- Graphic solutions
- Right triangle solutions
- Sine, cosine, tangent, ratios, functions of angles in degrees
- Interpolation, angles of elevation and depression
- Numeric applications